

## SAMPLING AND SIGNAL RECONSTRUCTION

### Performance Objectives

- A. Investigate methods used to sample a signal and to recognize the signal that results from each method.
- B. Investigate a method used to reconstruct the intelligence from a sampled signal and demonstrate how the sampling signal frequency affects reconstruction.

### Basic Concepts

1. Sampling is a method used in pulse modulation to identify the intelligence signal by a sequence of pulses that represents the intelligence at a particular time.
2. Natural sampling is a type of sampled signal in which the top of each sample pulse follows the intelligence signal during the pulse-width time of the sampling signal.
3. Flat-topped sampling is a type of sampled signal in which the top of each sample pulse represents a single level of intelligence during the pulse-width time of the sampling signal.
4. The sampling principle states that the intelligence can be reconstructed by filtering when the sampling signal frequency ( $f_s$ ) (sampling rate) is greater than twice the maximum intelligence signal frequency ( $f_m$ ).
5. The Nyquist rate is a condition that occurs when the sampling signal frequency is equal to twice the maximum intelligence signal frequency ( $f_s = 2f_m$  where  $f_s$  is the sampling signal frequency and  $f_m$  is the maximum frequency of the intelligence signal).
6. The frequency response of the low-pass filter must be capable of passing the maximum intelligence signal frequency while rejecting sideband frequencies of the sampling signal to reconstruct the intelligence free of distortion.

No single sampling signal frequency or frequency response of a low-pass filter network performs best to reconstruct the intelligence. It takes a combination of both.

Selecting a sampling signal frequency is defined by a principle. The sampling principle which applies to pulse modulation techniques states that to have the ability to reconstruct the intelligence signal, the sampling signal frequency ( $f_s$ ) must be greater than twice the maximum frequency ( $f_m$ ) of the intelligence signal ( $f_s > 2f_m$ ). This condition provides a sufficient number of samples to approximate the intelligence signal making it practical for a low-pass filter to reconstruct the intelligence with little or no distortion as shown in Figure 2-1. Typically most pulse modula-

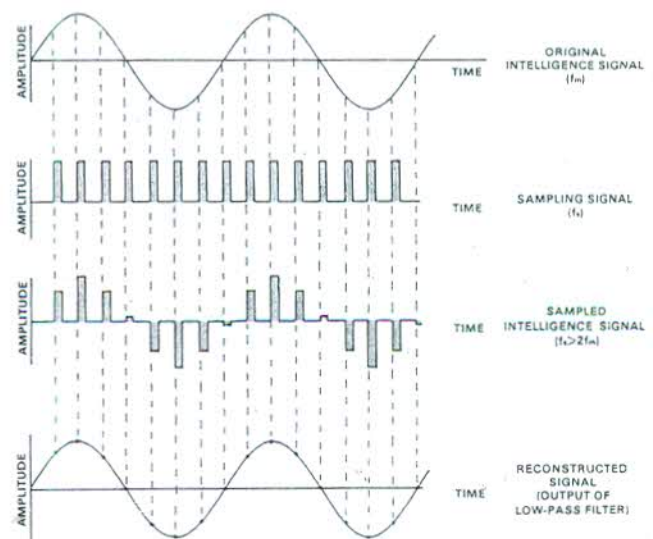


Figure 2-1

### Introductory Information

Sampling and signal reconstruction are two important considerations when transmitting and receiving information using pulse modulation techniques. The ability to reconstruct the intelligence depends on how often the original intelligence signal at the transmitter is sampled and how sharp the frequency response of a low-pass filter network is at the receiver.

tion sampling rates for speech over telephone channels, whose bandwidth is about 300 to 3kHz, is 8000 samples per second. The 8000 samples per second is slightly more than twice 3kHz which satisfies the sampling principle.

Sampling at a rate that is equal to or less than twice the maximum frequency of the intelligence signal should be avoided. Sampling at these rates will distort



the reconstructed intelligence signal. Figure 2-2 shows the results of insufficient sampling. The shape of the reconstructed signal has no resemblance of the original intelligence.

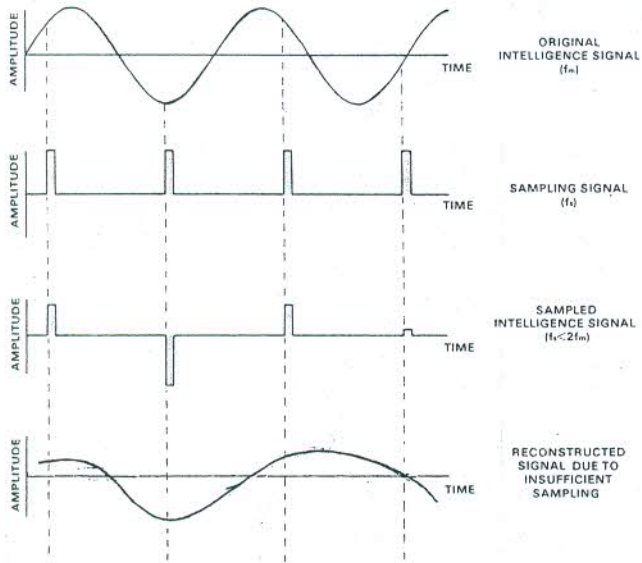


Figure 2-2

Signal reconstruction is the process of recovering intelligence from a sampled signal. In the receiver a low-pass filter filters the sampled signal and outputs the reconstructed intelligence that is an undistorted replica of the original intelligence. Two factors governing the ability to reconstruct the intelligence are the sampling signal frequency and frequency response of the low-pass filter. The sampling signal frequency must be greater than twice the maximum frequency of the intelligence. The cutoff frequency of the low-pass filter must be high enough to pass the maximum

intelligence signal frequency but low enough to reject sideband frequencies of the sampling signal frequency. Figure 2-3 shows the frequency spectrum (amplitude versus frequency) of a sampled intelligence signal. The frequency  $f_m$  is the maximum intelligence signal frequency that the low-pass filter must pass. The pairs of sideband frequencies centered around the sampling signal frequencies  $f_s$  ( $f_s - f_m$  and  $f_s + f_m$ ),  $2f_s$  ( $2f_s - f_m$  and  $2f_s + f_m$ ), and  $3f_s$  ( $3f_s - f_m$  and  $3f_s + f_m$ ), must be rejected by the low-pass filter. Provided the frequency response of the low-pass filter falls between frequencies  $f_m$  and  $f_s - f_m$  (shown by dashed lines) the reconstructed intelligence will be free of distortion. The reconstructed intelligence becomes distorted when the frequencies at and above  $f_s - f_m$  are allowed to pass through with the intelligence frequency  $f_m$ . When this occurs additional filtering or signal smoothing is required.

Additional Reading

See bibliography at the back of the manual for additional reading material related to this subject.

Equipment And Materials

- Power Source +15Vdc, 100mA
- Power Source -15Vdc, 100mA
- Electronic VOM
- Dual Trace Oscilloscope
- AF Generator
- Frequency Counter
- Pulse Modulation Trainer

Exercise Procedure

Objective A. Investigate methods used to sample a signal and to recognize the signal that results from each method.

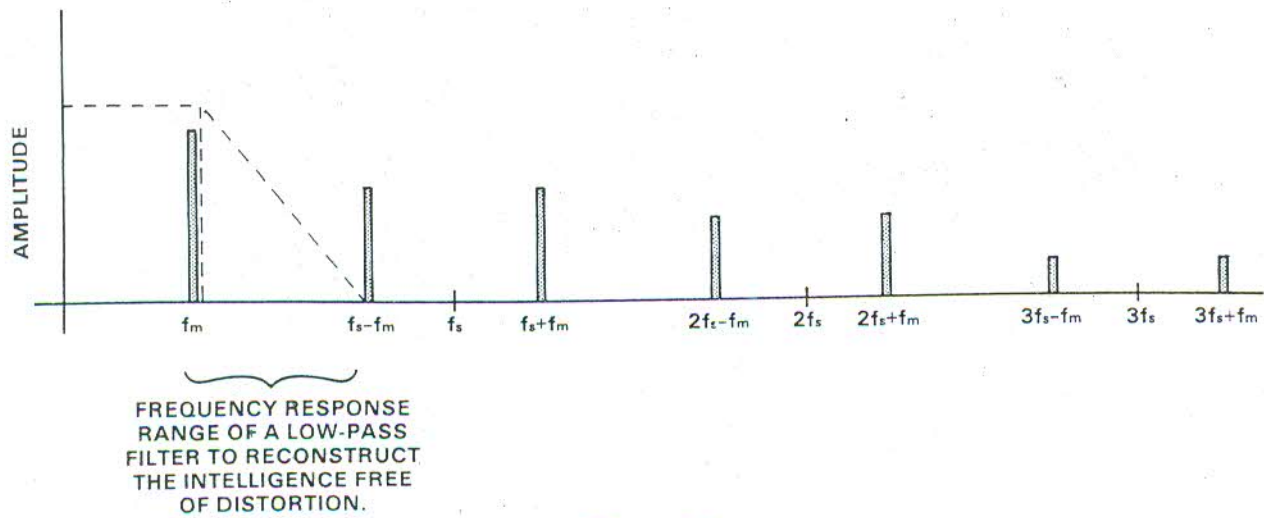


Figure 2-3



## Preparatory Information

Pulse amplitude, pulse width, and pulse position are three types of pulse modulation. The generation of each begins by identifying the intelligence signal with a sequence of pulses representing the intelligence at a particular time. A simple electronically-controlled switch shown in Figure 2-4 shows the concept used to sample the intelligence. A sampling signal operating at a low duty cycle controls the position of the switch.

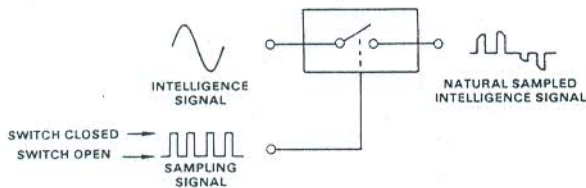


Figure 2-4

The switch is closed for the duration of each pulse which allows the intelligence at that sampling time to become part of the output. The switch is open for the remainder of each sampling period making the output zero. The signal generated by the method shown in Figure 2-4 is called a natural sampled signal.

Another form of a sampled signal that occurs in pulse modulation is called flat-top sampling. Figure 2-5

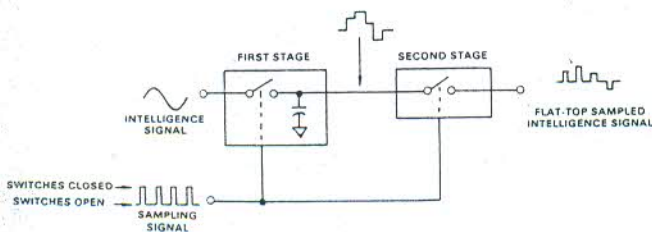


Figure 2-5

illustrates the concept. The first stage is used to sample and hold a level of intelligence. The capacitor charges to the level of the intelligence when the switch is closed and holds that value when the switch opens and until the switch closes again to take a new sample. The result is a staircase signal. The staircase signal is then input to the second stage which is identical to the circuit used to naturally sample the intelligence. The switch closes for the duration of each sampling signal pulse allowing a portion of each step to become part of the output. The switch is open for the remainder of each sampling period making the output zero. The resulting output is a flat-top sampled signal of the original intelligence.

The SAMPLER, SAMPLE/HOLD, and CLOCK circuits on the trainer are used to produce natural and flat-top sampling. A schematic of the SAMPLER and CLOCK circuit is shown in Figure 2-6. An intelligence signal input at J6 is ac coupled to the analog input of a sample and hold integrated circuit U5. The second input, at pin 8 (J9), represents the sampling signal input. The sampling signal is developed starting with the CLOCK circuit. A 555 IC timer U1 connected as an astable multivibrator outputs a frequency-adjustable pulse train at U1-3. Resistors R1, R2, R3 and capacitor C1 establish the timing. The pulse train from U1-3 is then input to a master/slave J-K flip flop. Each positive edge of the incoming pulses toggle the output of flip flop U2A-1. The result is a 50% duty cycle clock signal at J1. The J1 output is input to flip flop U2B-13 which divides the signal frequency by two, and outputs the signal at J3. The J1 output is the clock input to the sampler circuit at J7. The clock signal is buffered by inverters U4C and U4B, differentiated by resistor R12 and capacitor C10, and passed through inverter U4A to form the sampling signal. Diode CR5 provides a fast discharge for C10 and clamps the input voltage of U4A to +12Vdc during each cycle of the signal input at J7. The output of the sampler circuit is at J8.

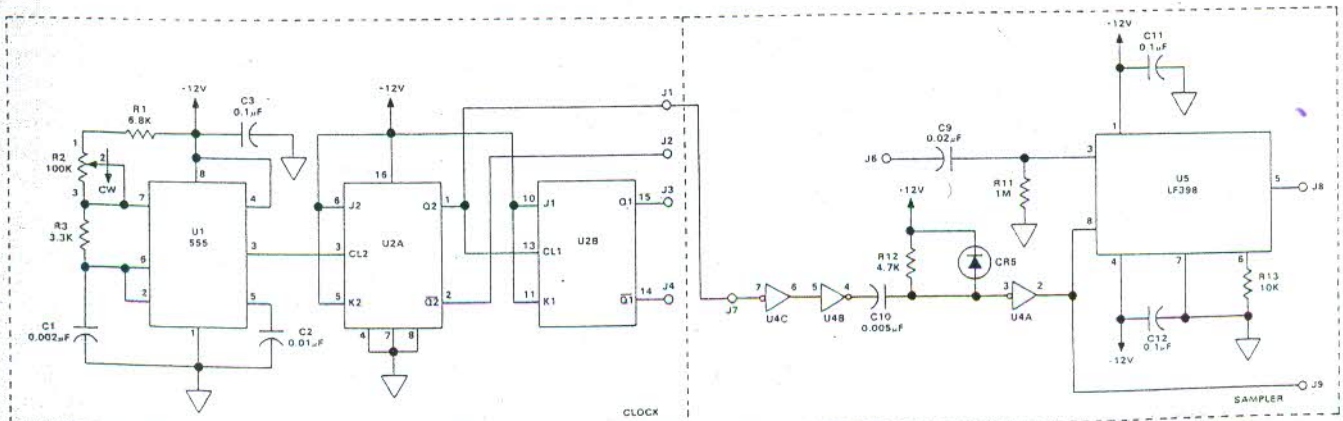


Figure 2-6



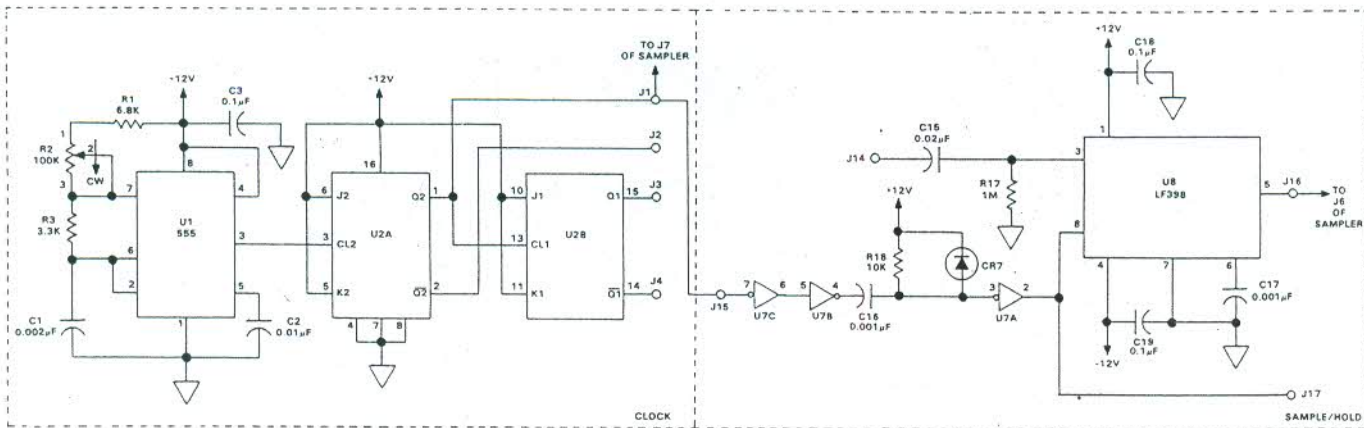


Figure 2-7

A schematic of the SAMPLE/HOLD circuit is shown in Figure 2-7. The circuitry is identical to the sampler circuit except capacitor C17 replaces resistor R13. The capacitor will charge to the level of the intelligence signal input at J14 at the instant of sampling and discharges slowly holding the output at that level for each sampling period. The sample/hold output at J16 is input to the sampler circuit at J6 (Figure 2-6) to produce a flat-top sampled signal representing the intelligence.

1. a) Connect a +15Vdc supply voltage across the +15V and GND jacks and a -15Vdc supply voltage across the -15V and GND jacks on the trainer.
- b) Connect the circuit shown in Figure 2-8. Turn the FREQ ADJ control on the CLOCK circuit fully counterclockwise. Turn the trainer power on.

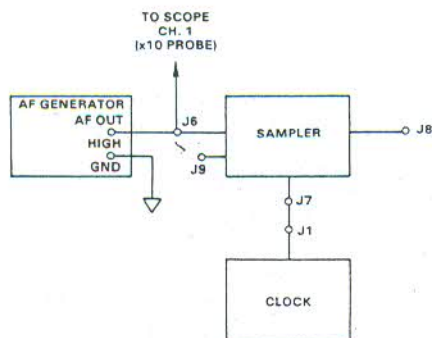


Figure 2-8

Set the sweep on the scope to 0.2ms/cm and trigger the scope on channel 1. Set the AF generator for a sine wave output of 3Vp-p at 1kHz. What does the signal input to J6 represent?

- c) Connect a second x10 probe from channel 2 of the oscilloscope to J8. Observe the output of the sampler circuit at J8 and carefully adjust the AF generator to stabilize the waveform.

**NOTE:** This adjustment is sensitive since the oscilloscope is displaying two unrelated frequencies.

Explain how the output at J8 is related to the intelligence signal input at J6.

- d) Move the scope probe connected at J6 to J9. Describe the signal and what it represents in the sampler circuit.

- e) Observe the sampling signal and output signal of the sampler circuit. Do the waveforms show when the intelligence signal input to J6 is sampled? Explain.

the output of the sample/hold circuit at J16. Trigger the scope on channel 1. Carefully adjust the AF generator to stabilize the waveform displayed on channel 2 (see note in step 1c). Explain the signal output at J16.

☐ f) Turn off power to the trainer.

☐ 2. a) Connect the circuit shown in Figure 2-9. Turn the FREQ ADJ control on the CLOCK circuit fully counterclockwise.

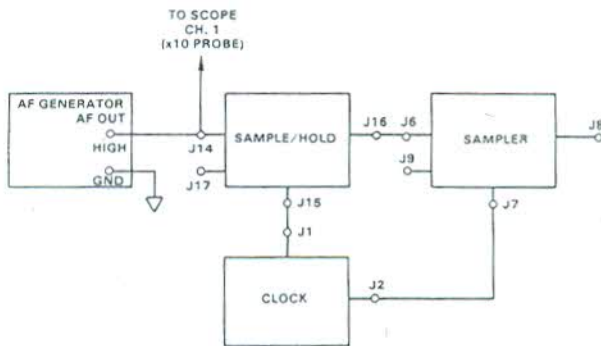


Figure 2-9

Set the AF generator for a sine wave output of 3Vp-p at 1kHz. Turn on power to the trainer. What does the signal input to J14 represent?

☐ d) Move the channel 1 probe connected at J14 to the output of the sampler circuit at J8. Move the channel 2 probe connected at J16 to the sampling signal input of the sampler circuit at J9. Identify and describe the signal at J8 and J9.

☐ b) Connect a second x10 scope probe from channel 2 of the oscilloscope to J17. Trigger the oscilloscope on channel 1. Describe the signal observed and explain its purpose.

☐ c) Turn off power to all equipment



**Objective B.** Investigate a method used to reconstruct the intelligence from a sampled signal and demonstrate how the sampling signal frequency affects reconstruction.

### Preparatory Information.

Reconstruction is the process of recovering the original intelligence from a sampled signal representing the intelligence that is modulated and output from a transmitter. After demodulating the signal at the receiver a low-pass filter can be used to recover the intelligence provided certain conditions are met. Beside conditions required to modulate the sampled signal for transmission and to demodulate the sampled signal at the receiver, the original intelligence must be sampled at an acceptable rate and the frequency response of the low-pass filter must be sharp enough to pass the intelligence signal frequencies and reject all others.

The frequency of the sampling signal used to sample the original intelligence is one condition that affects

reconstruction. The sampling signal frequency must be selected such that enough samples are taken to provide a good approximation of the original intelligence. Selection of the sampling signal frequency is controlled by a sampling principle. The sampling principle states that the original intelligence must be sampled at a rate at least two times greater than the highest frequency of the original intelligence to be reconstructed by low-pass filtering ( $f_s \geq 2f_m$ ).

The sampling signal frequency is  $f_s$ . The maximum frequency of the intelligence signal is  $f_m$ . In practice, since a low-pass filter is not ideal and does not have a sharp frequency response, the sampling signal frequency selected is never equal to but greater than twice the maximum intelligence frequency. For example a voice-grade signal with a maximum frequency of 3kHz may require a sampling signal frequency of 8kHz or higher depending on the frequency response of the low-pass filter.

The frequency response characteristics required of a low-pass filter to reconstruct the intelligence can be shown by a frequency spectrum diagram. Figure 2-10

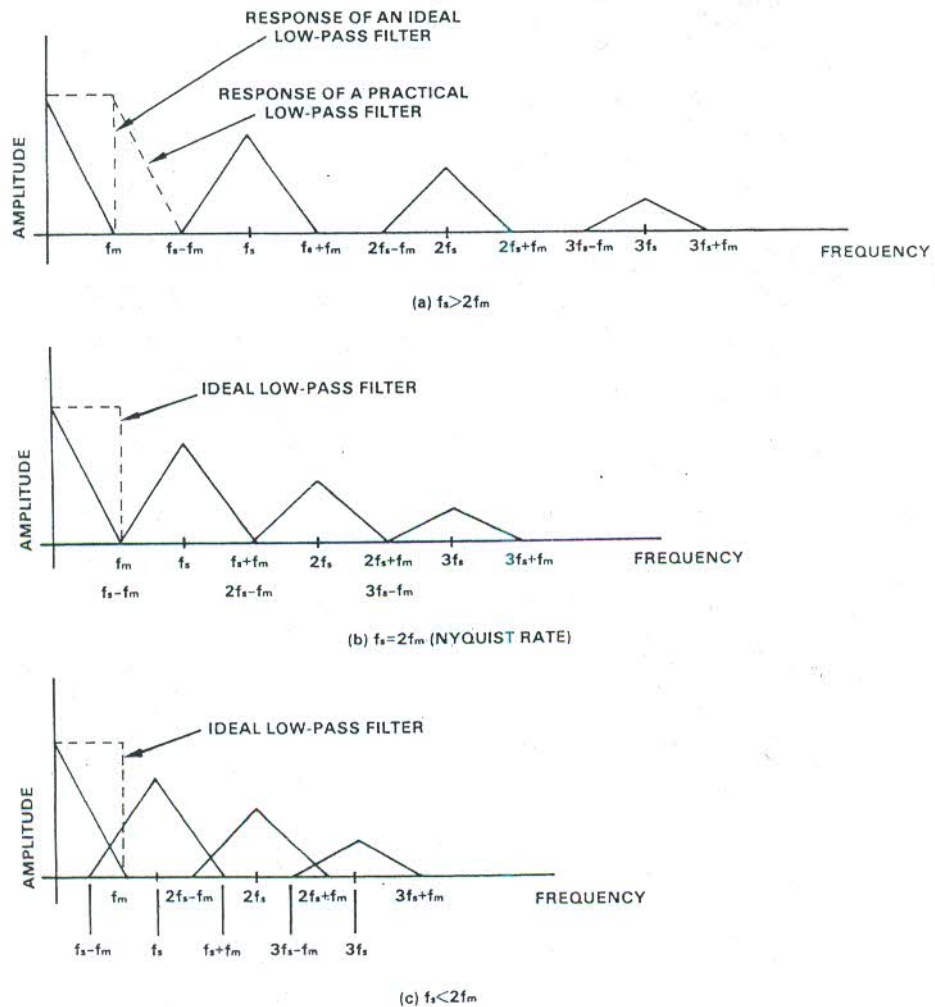


Figure 2-10

shows the spectrum of intelligence signals with a maximum frequency  $f_m$  when sampled with different frequencies  $f_s$ . The spectrum in (a) satisfies the sampling principle  $f_s > 2f_m$ , in (b)  $f_s = 2f_m$  (the Nyquist rate), and in (c)  $f_s < 2f_m$ . Each spectrum shows the maximum frequency of the intelligence signal and pairs of upper and lower sideband frequencies centered around the sampling signal frequency at  $f_s$ ,  $2f_s$ ,  $3f_s$ ... The low-pass filter must completely reject sideband frequencies of the sampling signal to reconstruct the intelligence without distortion.

The condition shown in Figure 2-10a is most practical for a low-pass filter to reconstruct the intelligence. The reconstructed intelligence will be free of distortion when the frequency response of the low-pass filter (shown by dashed lines) rolls off to zero after  $f_m$  and before the lower sideband of  $f_s$  at  $f_s - f_m$ . Figure 2-11a shows the results of reconstruction when the conditions of Figure 2-10a exist. The condition in Figure 2-10b requires an *ideal* low-pass filter which is not practical. The sampling signal frequency causes the lower sideband frequency  $f_s - f_m$  to be equal to  $f_m$ . The reconstructed intelligence will be distorted. Figure 2-11b shows a typical distorted intelligence signal at the Nyquist rate. The condition in Figure 2-10c is called *aliasing*. Aliasing occurs when sideband frequencies of the sampling signal normally outside of the intelligence signal frequency band overlap. The sampled signal does not approximate the original intelligence and the low-pass filter reconstructs an entirely different signal. Figure 2-11c shows the results of reconstruction when the conditions of Figure 2-10c exist.

A schematic of the circuitry used on the trainer to demonstrate reconstruction is shown in Figure 2-12. Two low-pass filters are cascaded (in series) and used to reconstruct the intelligence from a sampled signal output from the sampler circuit at J8. The development of the sampler circuit output is identical to the

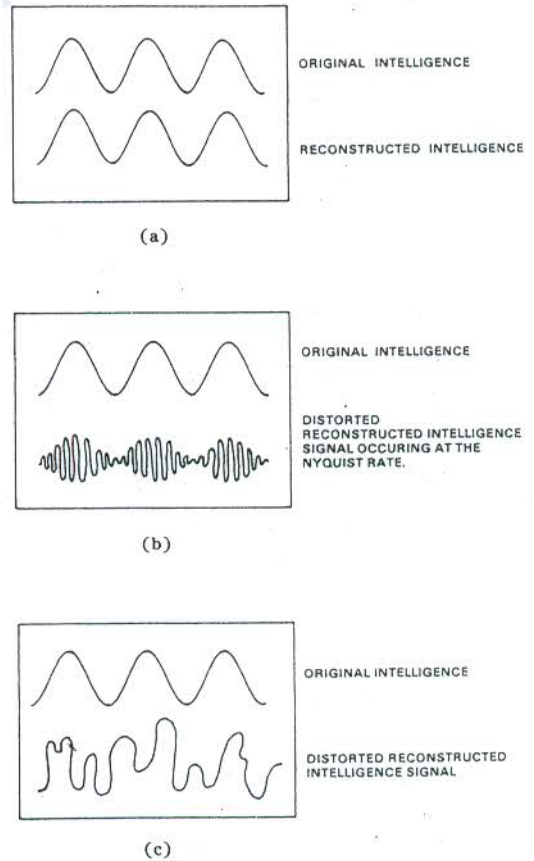
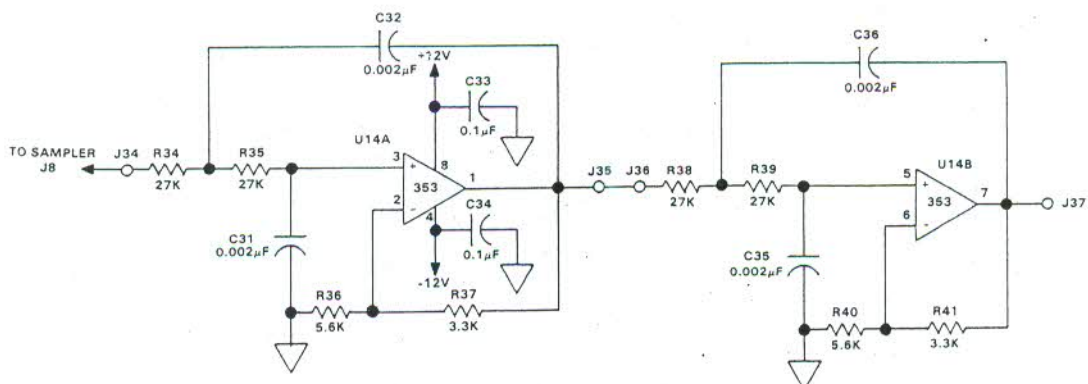


Figure 2-11

operation described in Objective A of this Laboratory Exercise.

The frequency response of the first non-inverting active low-pass filter circuit is determined by resistors R34 and R35 and capacitors C31 and C32. This multiple RC section provides a sharper frequency response than a single-section filter with one RC combination. Resistors R36 and R37 set the gain of the





operational amplifier U14A which is about 1.5 (or  $1 + R37/R36$ ). Capacitors C33 and C34 are decoupling capacitors. The input to the low-pass filter is J34. The output is J35. The cutoff frequency is about 3kHz.

The frequency response of the second non-inverting active low-pass filter circuit is determined by resistors R38 and R39 and capacitors C35 and C36. Resistors R40 and R41 set the gain of the operational amplifier U14B which is about 1.5 (or  $1 + R41/R40$ ). The input to the low-pass filter is J36. The output is J37. The cutoff frequency is about 3kHz.

Cascading the two low-pass filters improves the frequency response. The cutoff frequency remains at 3kHz but with a sharper rolloff.

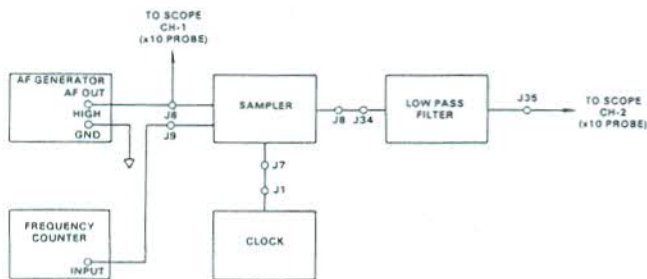


Figure 2-13

☐ 3. a) Connect the circuit shown in Figure 2-13. Turn the FREQ ADJ control on the CLOCK circuit fully clockwise. Turn on power to the trainer and set the AF generator to 3kHz at 3Vp-p. What does the signal output from the low-pass filter at J35 (displayed on channel 2 of the oscilloscope) represent?

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☐ b) Observe the reconstructed intelligence and the original intelligence. Is there distortion in the reconstructed intelligence signal?

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☐ c) Record the sampling signal frequency with distortion in the reconstructed intelligence signal.

$$f_s = \text{..... kHz}$$

How does the sampling signal frequency compare with the Nyquist rate?

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☐ d) Observe the reconstructed intelligence signal on the oscilloscope. What reason can be given to indicate why the reconstructed intelligence output from the low-pass filter at J35 contains some distortion when  $f_s > 4f_m$ ?

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☐ e) Connect the second low-pass filter in cascade with the first (connect a jumper wire from J35 to J36). Connect the channel 2 probe to J37. Observe the reconstructed intelligence signal at J37. What effect does cascading the second low-pass filter produce on the reconstructed intelligence? Explain.

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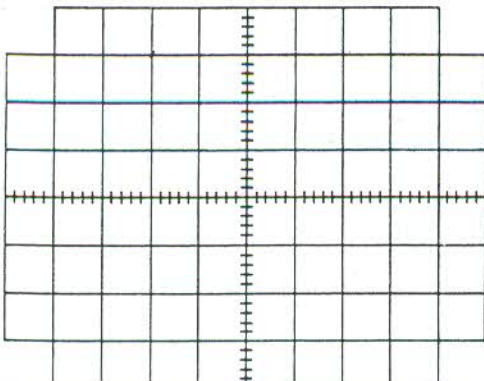


- ☐ f) Slowly rotate the FREQ ADJ control counterclockwise while observing the reconstructed intelligence signal. What sampling signal frequency starts to distort the reconstructed intelligence signal?

$f_s = \dots \text{ kHz}$

- ☐ g) Use the value of  $f_s$  recorded in step f to determine how it compares with the Nyquist rate and the value recorded in step c. Relate the  $f_s$  values to the resulting reconstructed intelligence and filter circuitry used. What can you conclude?

- ☐ h) Move the channel 1 probe connected at J6 to J35 to view the reconstructed intelligence output from the first low-pass filter. Set the sweep rate to 4ms/cm. Rotate the FREQ ADJ control to set the sampling signal frequency at 6kHz. Sketch the signals.



What rate is the original intelligence being sampled?

- ☐ i) Rotate the FREQ ADJ control to set the sampling signal frequency at 5kHz to place the lower sideband frequency of  $f_s$  below  $f_m$ . What condition does this create? Explain.

- ☐ j) The sampling signal frequency is 5kHz. The maximum frequency of the intelligence signal is 3kHz. What is the lower sideband frequency of  $f_s$ ?

- ☐ k) Turn off power to all equipment.

### Summary

In this laboratory exercise you demonstrated a method used to reconstruct the intelligence from a sampled signal. You determined that the sampling signal frequency and frequency response of the low-pass filter network had a direct influence on reconstructing the intelligence. A sampling signal frequency greater than twice the maximum frequency of the intelligence signal ( $f_s > 2f_m$ ) and a low-pass filter with a sharp frequency response can reconstruct the intelligence free of distortion. Sampling the intelligence at the Nyquist rate ( $f_s = 2f_m$ ) or less than twice the maximum intelligence signal frequency ( $f_s < 2f_m$ ) will distort the reconstructed intelligence. The distortion is caused by an insufficient sampling rate (sampling signal frequency) which fails to provide a good approximation of the original intelligence.

**Quiz**

1. The amplitude of the sample pulses follows the intelligence when the intelligence signal is
  - a. reconstructed.
  - b. free of distortion.
  - c. aliased.
  - d. natural sampled.
2. Flat-top sampling refers to the sampled signals
  - a. frequency.
  - b. duty cycle.
  - c. pulse amplitude.
  - d. distortion.
3. Satisfying the condition that the sampling signal frequency ( $f_s$ ) must be greater than twice the maximum intelligence signal frequency ( $f_m$ ) to reconstruct the intelligence by filtering
  - a. is a statement of the sampling principle.
  - b. creates a condition known as aliasing.
  - c. is known as the Nyquist rate.
  - d. places the lower sideband frequency of  $f_s$  at  $f_m$ .
4. What is the lower sideband frequency of  $f_s$  when the maximum intelligence signal frequency is  $f_m$ ?
  - a.  $f_m + f_s$
  - b.  $2f_s - f_m$
  - c.  $f_s - f_m$
  - d.  $3f_s + f_m$
5. The Nyquist rate is expressed as
  - a.  $f_s = f_m$ .
  - b.  $f_s < f_m$ .
  - c.  $f_s = 2f_m$ .
  - d.  $f_s > 2f_m$ .
6. Aliasing
  - a. occurs when sideband frequencies are shifted into the intelligence frequency band.
  - b. will not produce distortion in the reconstructed signal.
  - c. occurs when the sampling signal frequency is greater than twice the maximum intelligence signal frequency.
  - d. is a condition when two low-pass filters must be cascaded to remove distortion from the reconstructed intelligence.